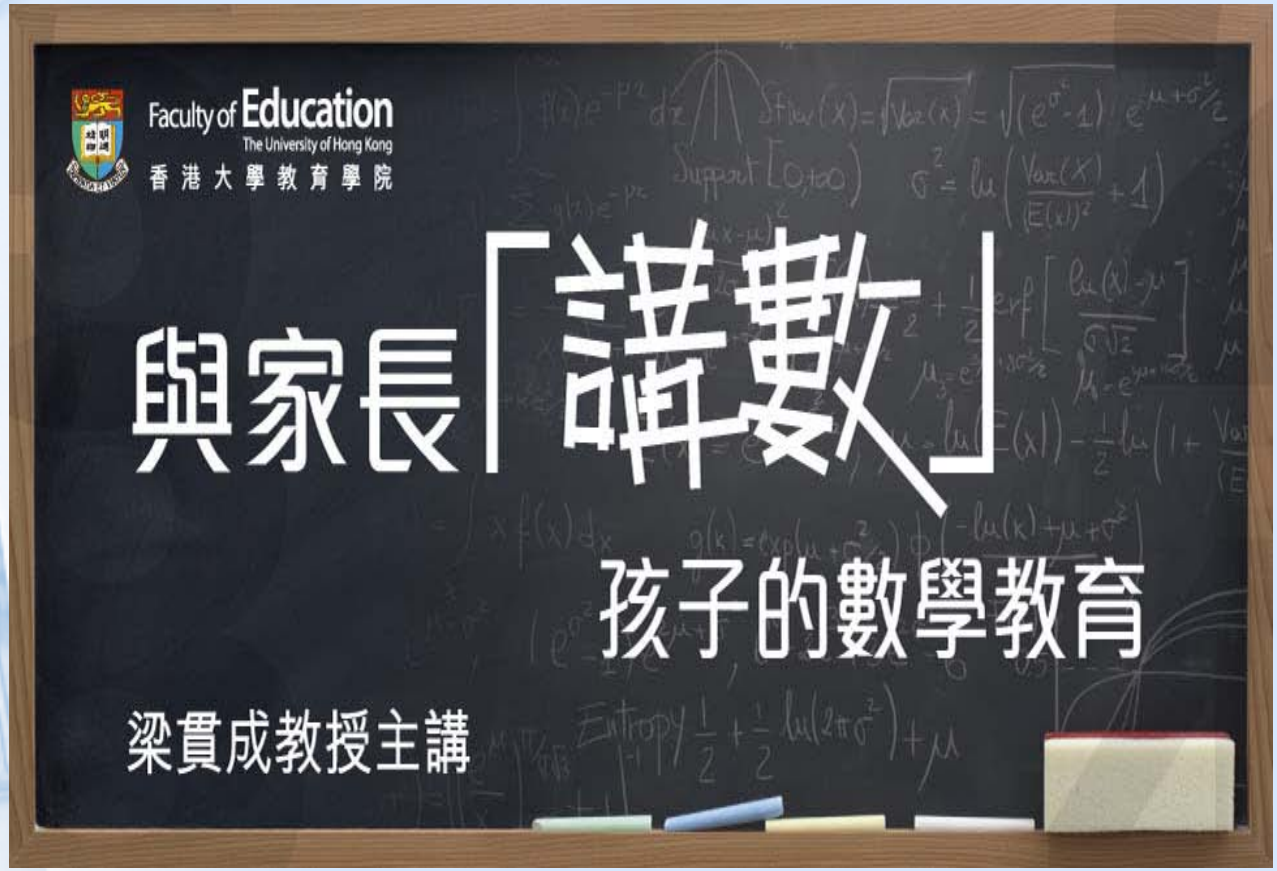

Faculty of Education
The University of Hong Kong
香港大學教育學院
與家長「講數」
孩子的數學教育
梁貫成教授主講

大綱

1. 為何要「講數」?
2. 數學是甚麼?
3. 數學有甚麼用?
4. 今天的數學課與昨天的數學課
5. 如何協助孩子學習數學?
6. 如何培養孩子對數學的興趣?

為何要「講數」?

- 香港的數學教育出了問題麼?
- 香港學生在國際數學測試的表現
  - 國際數學及科學趨勢研究(TIMSS)
  - 國際學生評量方案 (OECD PISA)
- 為何學生覺得數學難、悶?
  - 「街頭數學」研究的啟示
- **數學學習要對孩子有意義**

數學是甚麼?

- 探究「量」與「形」及其規律與關係的學科
- 「準確，精密的思維(與表達) 」
- 數學 ≠ 計算 (數學 ⊃ 計算)
- 數學是結果 (代數,幾何等) 抑或是過程 (解題,証明等)?
- 數學(結果)的性質
  - 抽象
  - 嚴謹
- 數學過程
  - 從具體經驗到抽象思維
  - 從隨意表達到嚴謹思維與語言

數學有甚麼用?

- 三個問題:
  - 唸數學將來有何出路?
  - 數學在日常生活中有甚麼應用?
  - 「我所學的數學有何意義」?

- 有何出路?

「*書中自有黃金屋*」(宋真宗 趙恒)

- 與數學有關的行業：精算師,金融分析員, 動畫電腦程式員, 教師(?)
- 華爾街日報2009年的調查報告
- 作為其他學科、行業的基礎

數學在日常生活中的應用

數學有用論

- 丘成桐 訪問 *(Discover Magazine)*

數學無用論

- 你曾否在日常生活中運用過二次方程解決現實問題?
- 在日常生活中,加減乘除,再加上一點统計和概率,已經足够有餘

「*數學有何用?*」- 歐幾里德 (Euclid) 的答案

「我所學的數學有何意義」？

「*人生在世有幾何？何必苦苦學幾何？學了幾何能幾何？不學幾何又幾何？*」

- 明白數學就覺得有何意義

<u>數學教育的宗旨及目標</u>

- 應用於其他學科
- 思維訓練
- 人類文化遺產
- 作為有學識公民的裝備
- 享受數學

今天的數學課與昨天的數學課

- 五十年不變?
- 活動教學, 課堂互動較強
- 較多開放型的數學題
- 較多數學情境, 較少數學語言描述
- 電腦軟件對數學的教與學(將會)有甚麼影響?

如何協助孩子學好數學?

- 「*題海戰術*」、「*熟能生巧*」?
- 何謂好? 名列前茅? 快而準?
- 「*求學不是求分數*」
- 如何扼殺創新力?
- 有質素的練習(質素 - 練習的內容與練習者的態度)
- 「*學而不思則罔*」《論語·為政》
- 學習態度非常重要
  - 你希望孩子對數學學習抱有甚麼態度?
  - 「*固本培元*」抑或 即食文化?

該讓孩子參加輔助課程嗎?

- 「*存在即合理*」?《黑格爾》
- 合理≠對、好
- 不存在(於學校課程)即不合理?
- 應否讓孩子參加:
  - 補習
  - 公文數
  - 珠心算
  - 奧數

我自己可協助孩子學數學嗎?

- *怎樣解題?* (波利亞)
  (*How to Solve It?*, Polya)
  1. 弄清問題
  2. 擬定計劃
  3. 實現計劃
  4. 回顧
- 元認知 (*梁貫成: 你在做甚麼?*)
  「*知之為知之*」新解

「*知之為知之, 不知為不知, 是知也*」- 《論語 · 為政》

- 良好的工作習慣 (如:認真、專注、小心、抽離)
- 孩子也要多「*講數*」

如何培養孩子對數學的興趣?

「*知之者不如好之者，好之者不如樂之者。*」《論語．雍也第六》

- 你自己沒有的, 不能给你的孩子
  - 與孩子培養共同興趣
- 先培養(或:不要扼殺)好奇心,留意周圍與數學有關的事物
  - 為甚麼?你如何知道?有何分別?大概多少?
- 多给孩子空間

親子數學活動

- 多與孩子溝通
  - 告訴我,考考我
- 數學遊戲
- 閱讀與數學有關的網頁及書
  - 台灣 考聯經出版社出版《*親子數學*》
  - 台灣 九章出版社
  - Jo Boaler
  - Ian Stewart
  - Timothy Gowers: “Mathematics: A very short introduction”

謝謝!

香港大學 教育學院
梁貫成
(電郵: frederickleung@hku.hk)

Table 1.1

Distributions of Mathematics Achievement - Upper Grade (Fourth Grade*)

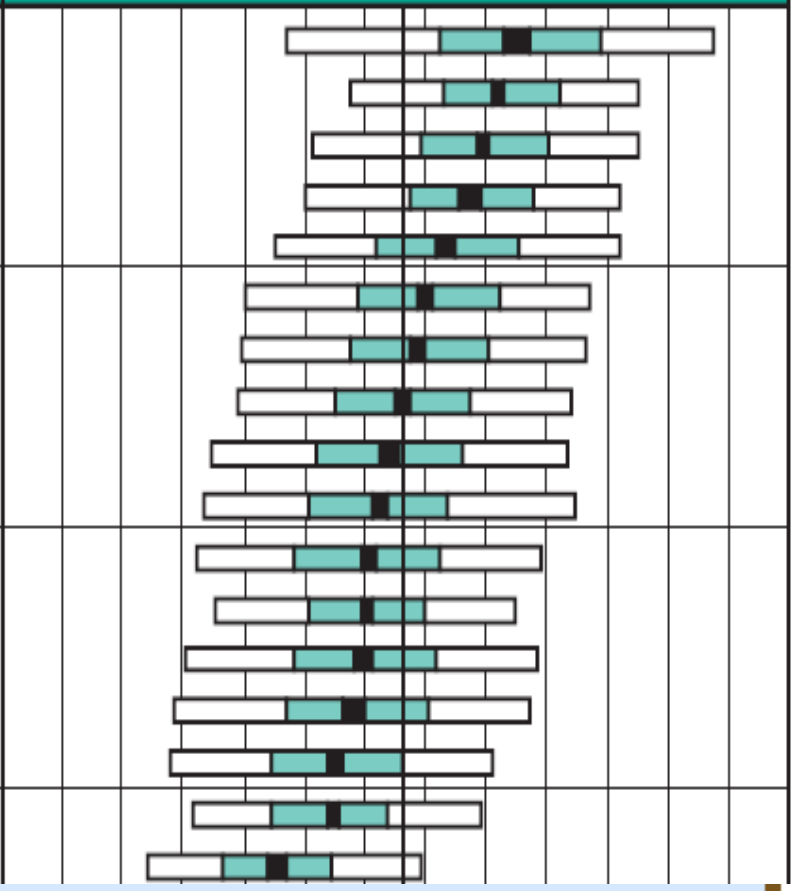
Country	Mean	Years of Formal Schooling	Average Age	Mathematics Achievement Scale Score
Singapore	625 (5.3)	4	10.3	
Korea	611 (2.1)	4	10.3	
Japan	597 (2.1)	4	10.4	
Hong Kong	587 (4.3)	4	10.1	
Czech Republic	567 (3.3)	4	10.4	
Ireland	550 (3.4)	4	10.3	
United States	545 (3.0)	4	10.2	
Canada	532 (3.3)	4	10.0	
† Scotland	520 (3.9)	5	9.7	
†2 England	513 (3.2)	5	10.0	
Cyprus	502 (3.1)	4	9.8	
Norway	502 (3.0)	3	9.9	
New Zealand	499 (4.3)	4.5–5.5	10.0	
Greece	492 (4.4)	4	9.6	
Portugal	475 (3.5)	4	10.4	
Iceland	474 (2.7)	4	9.6	
Iran, Islamic Rep.	429 (4.0)	4	10.5	

Table 1.1

Distributions of Mathematics Achievement - Upper Grade (Eighth Grade*)

Country	Mean	Years of Formal Schooling	Average Age	Mathematics Achievement Scale Score
Singapore	643 (4.9)	8	14.5	
Korea	607 (2.4)	8	14.2	
Japan	605 (1.9)	8	14.4	
Hong Kong	588 (6.5)	8	14.2	
† Belgium (Fl)	565 (5.7)	8	14.1	
Czech Republic	564 (4.9)	8	14.4	
Slovak Republic	547 (3.3)	8	14.3	
1 Switzerland	545 (2.8)	7 or 8	14.2	
France	538 (2.9)	8	14.3	
Hungary	537 (3.2)	8	14.3	
Russian Federation	535 (5.3)	7 or 8	14.0	
Ireland	527 (5.1)	8	14.4	
Canada	527 (2.4)	8	14.1	
Sweden	519 (3.0)	7	13.9	
New Zealand	508 (4.5)	8.5 - 9.5	14.0	
†2 England	506 (2.6)	9	14.0	
Norway	503 (2.2)	7	13.9	
† United States	500 (4.6)	8	14.2	
1 Latvia (LSS)	493 (3.1)	8	14.3	
Spain	487 (2.0)	8	14.3	
Iceland	487 (4.5)	8	13.6	
1 Lithuania	477 (3.5)	8	14.3	
Cyprus	474 (1.9)	8	13.7	
Portugal	454 (2.5)	8	14.5	
Iran, Islamic Rep.	428 (2.2)	8	14.6	

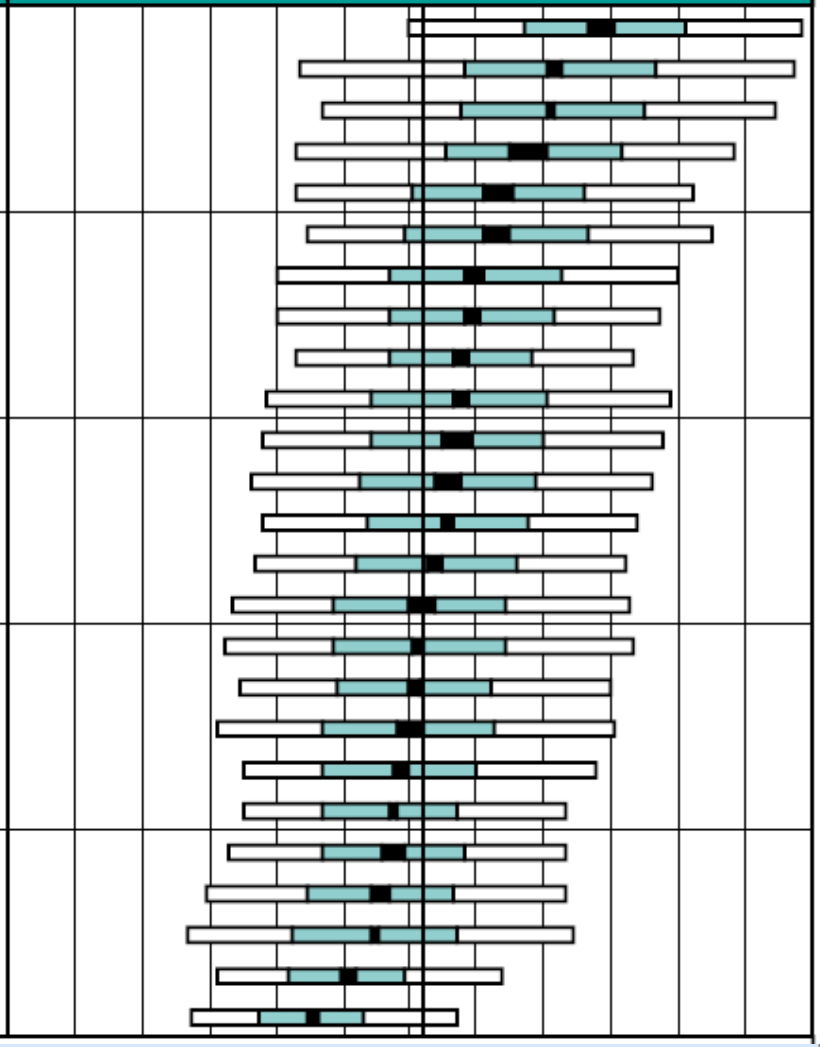


Exhibit 1.1 Distribution of Mathematics Achievement

TIMSS 1999 8th grade Mathematics

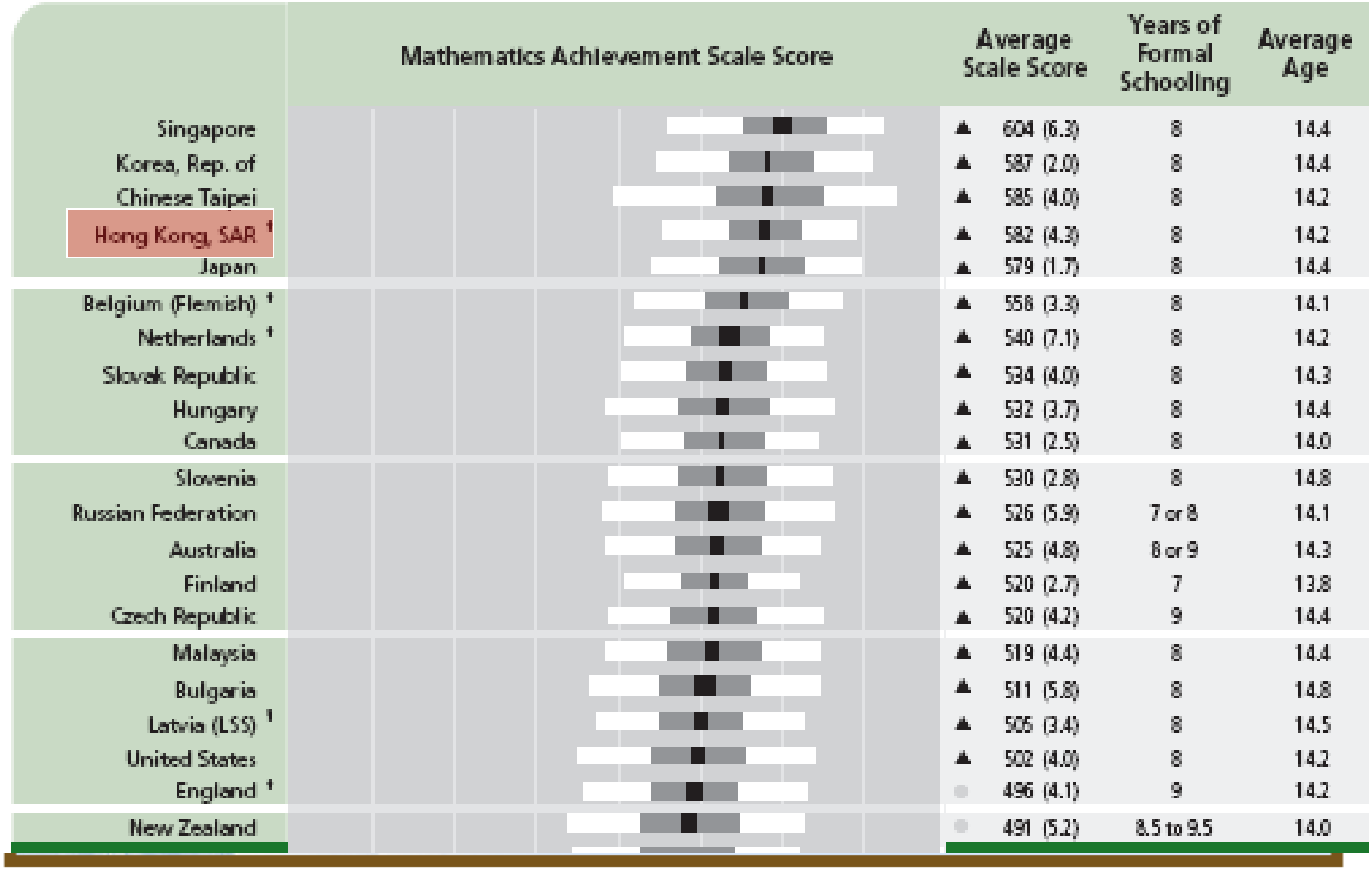


Exhibit 1.1: Distribution of Mathematics Achievement

MATHEMATICS Grade 4

Countries	Years of Schooling*	Average Age	Mathematics Achievement Distribution	Average Scale Score	Human Development Index**
Singapore	4	10.3		594 (5.6)	0.884
† Hong Kong, SAR	4	10.2		575 (3.2)	0.889
Japan	4	10.4		565 (1.6)	0.932
Chinese Taipei	4	10.2		564 (1.8)	–
Belgium (Flemish)	4	10.0		551 (1.8)	0.937
† Netherlands	4	10.2		540 (2.1)	0.938
Latvia	4	11.1		536 (2.8)	0.811
† Lithuania	4	10.9		534 (2.8)	0.824
Russian Federation	3 or 4	10.6		532 (4.7)	0.779
† England	5	10.3		531 (3.7)	0.930
Hungary	4	10.5		529 (3.1)	0.837
† United States	4	10.2		518 (2.4)	0.937
Cyprus	4	9.9		510 (2.4)	0.891
Moldova, Rep. of	4	11.0		504 (4.9)	0.700
Italy	4	9.8		503 (3.7)	0.916
† Australia	4 or 5	9.9		499 (3.9)	0.939
International Avg.	**4**	**10.3**		**495 (0.8)**	**–**
New Zealand	4.5 - 5.5	10.0		493 (2.2)	0.917
† Scotland	5	9.7		490 (3.3)	0.930

SOURCE: IEA's Trends in International Mathematics and Science Study (TIMSS) 2003

Exhibit 1.1: Distribution of Mathematics Achievement

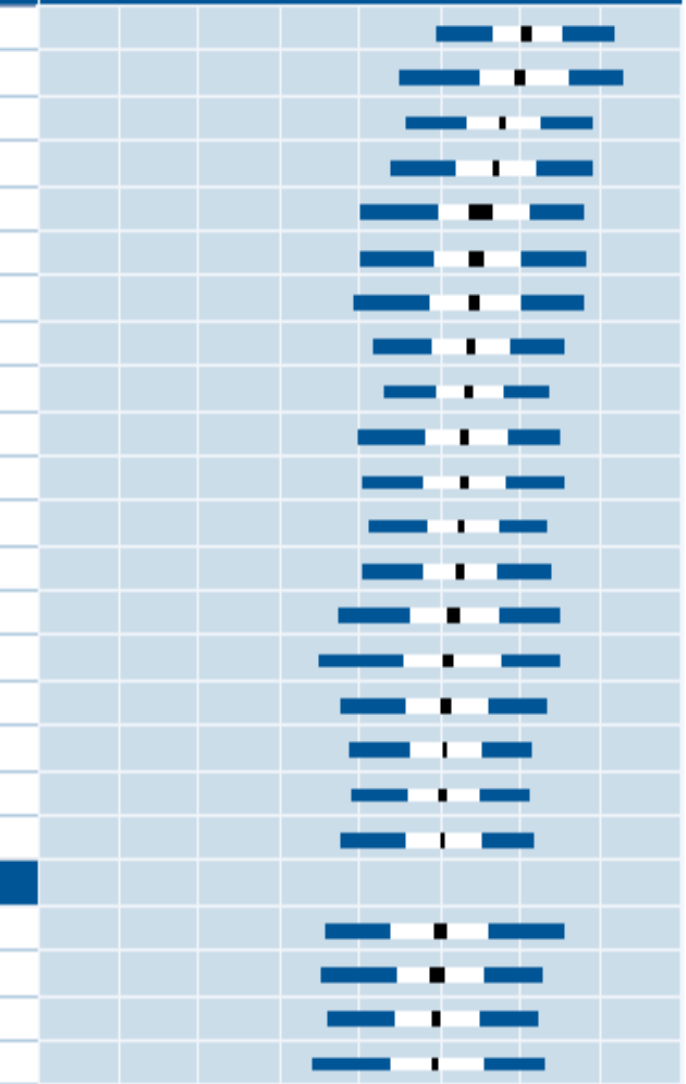
MATHEMATICS Grade 8

Countries	Years of Schooling*	Average Age	Mathematics Achievement Distribution	Average Scale Score	Human Development Index**
Singapore	8	14.3		605 (3.6)	0.884
◂◂ Korea, Rep. of	8	14.6		589 (2.2)	0.879
† Hong Kong, SAR	8	14.4		586 (3.3)	0.889
Chinese Taipei	8	14.2		585 (4.6)	–
Japan	8	14.4		570 (2.1)	0.932
Belgium (Flemish)	8	14.1		537 (2.8)	0.937
† Netherlands	8	14.3		536 (3.8)	0.938
Estonia	8	15.2		531 (3.0)	0.833
Hungary	8	14.5		529 (3.2)	0.837
Malaysia	8	14.3		508 (4.1)	0.790
Latvia	8	15.0		508 (3.2)	0.811
Russian Federation	7 or 8	14.2		508 (3.7)	0.779
Slovak Republic	8	14.3		508 (3.3)	0.836
Australia	8 or 9	13.9		505 (4.6)	0.939
‡ United States	8	14.2		504 (3.3)	0.937
1 Lithuania	8	14.9		502 (2.5)	0.824
Sweden	8	14.9		499 (2.6)	0.941
† Scotland	9	13.7		498 (3.7)	0.930
2 Israel	8	14.0		496 (3.4)	0.905
New Zealand	8.5 - 9.5	14.1		494 (5.3)	0.917
Slovenia	7 or 8	13.8		493 (2.2)	0.881
Italy	8	13.9		484 (3.2)	0.916
Armenia	8	14.9		478 (3.0)	0.729
1 Serbia	8	14.9		477 (2.6)	–
Bulgaria	8	14.9		476 (4.3)	0.795
Romania	8	15.0		475 (4.8)	0.773
International Avg.	**8**	**14.5**		**467 (0.5)**	**–**

SOURCE: IEA's Trends in International Mathematics and Science Study (TIMSS) 2003

Exhibit 1.1 TIMSS 2007 Distribution of Mathematics Achievement

TIMSS2007 Mathematics 4th Grade

Country	Mathematics Achievement Distribution	Average Scale Score	Years of Formal Schooling*	Average Age at Time of Testing	Human Development Index**
Hong Kong SAR		▲ 607 (3.6)	4	10.2	0.937
Singapore		▲ 599 (3.7)	4	10.4	0.922
Chinese Taipei		▲ 576 (1.7)	4	10.2	0.932
Japan		▲ 568 (2.1)	4	10.5	0.953
1 Kazakhstan		▲ 549 (7.1)	4	10.6	0.794
Russian Federation		▲ 544 (4.9)	4	10.8	0.813
England		▲ 541 (2.9)	5	10.2	0.946
1 Latvia		▲ 537 (2.3)	4	11.0	0.855
‡ Netherlands		▲ 535 (2.1)	4	10.2	0.953
1 Lithuania		▲ 530 (2.4)	4	10.8	0.862
2 † United States		▲ 529 (2.4)	4	10.3	0.951
Germany		▲ 525 (2.3)	4	10.4	0.935
† Denmark		▲ 523 (2.4)	4	11.0	0.949
Australia		▲ 516 (3.5)	4	9.9	0.962
Hungary		▲ 510 (3.5)	4	10.7	0.874
Italy		▲ 507 (3.1)	4	9.8	0.941
Austria		▲ 505 (2.0)	4	10.3	0.948
Sweden		503 (2.5)	4	10.8	0.956
Slovenia		502 (1.8)	4	9.8	0.917
TIMSS Scale Avg.		**500**			
Armenia		500 (4.3)	4	10.6	0.775
Slovak Republic		496 (4.5)	4	10.4	0.863
† Scotland		▼ 494 (2.2)	5	9.8	0.946
New Zealand		▼ 492 (2.3)	4.5 – 5.5	10.0	0.943

SOURCE: IEA's Trends in International Mathematics and Science Study (TIMSS) 2007

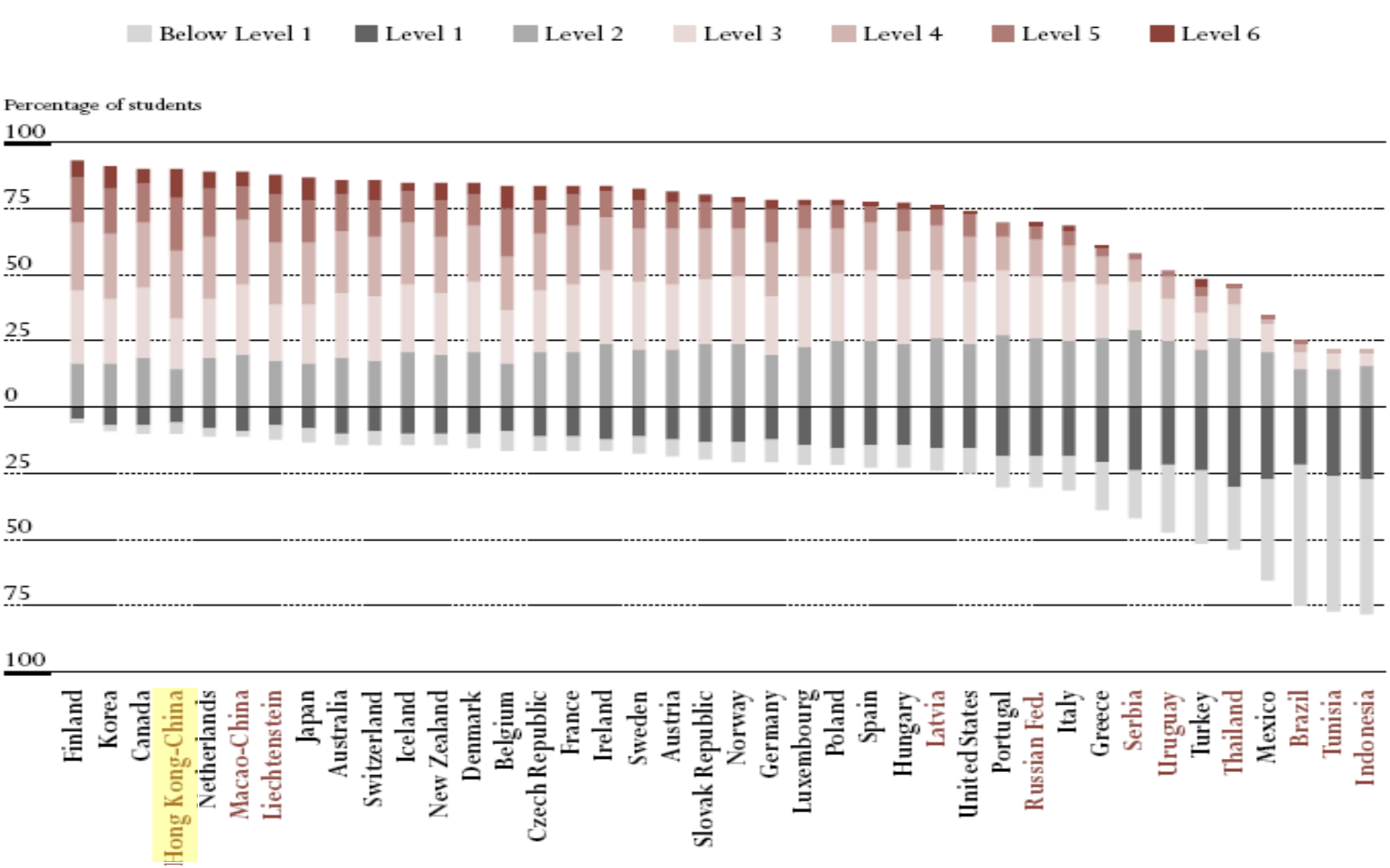
Exhibit 1.1 TIMSS 2007 Distribution of Mathematics Achievement (Continued)

TIMSS2007 Mathematics 8th Grade

Country	Mathematics Achievement Distribution	Average Scale Score	Years of Formal Schooling*	Average Age at Time of Testing	Human Development Index**
Chinese Taipei		▲ 598 (4.5)	8	14.2	0.932
Korea, Rep. of		▲ 597 (2.7)	8	14.3	0.921
Singapore		▲ 593 (3.8)	8	14.4	0.922
† Hong Kong SAR		▲ 572 (5.8)	8	14.4	0.937
Japan		▲ 570 (2.4)	8	14.5	0.953
Hungary		▲ 517 (3.5)	8	14.6	0.874
† England		▲ 513 (4.8)	9	14.2	0.946
Russian Federation		▲ 512 (4.1)	7 or 8	14.6	0.802
2 † United States		▲ 508 (2.8)	8	14.3	0.951
1 Lithuania		▲ 506 (2.3)	8	14.9	0.862
Czech Republic		504 (2.4)	8	14.4	0.891
Slovenia		501 (2.1)	7 or 8	13.8	0.917
TIMSS Scale Avg.		500			
Armenia		499 (3.5)	8	14.9	0.775
Australia		496 (3.9)	8	13.9	0.962
Sweden		▼ 491 (2.3)	8	14.8	0.956
Malta		▼ 488 (1.2)	9	14.0	0.878
† Scotland		▼ 487 (3.7)	9	13.7	0.946
1 2 Serbia		▼ 486 (3.3)	8	14.9	0.810
Italy		▼ 480 (3.0)	8	13.9	0.941
Malaysia		▼ 474 (5.0)	8	14.3	0.811
Norway		▼ 469 (2.0)	8	13.8	0.968
Cyprus		▼ 465 (1.6)	8	13.8	0.903

SOURCE: IEA's Trends in International Mathematics and Science Study (TIMSS) 2007

Figure 2.16a ■ **Percentage of students at each level of proficiency on the mathematics scale**

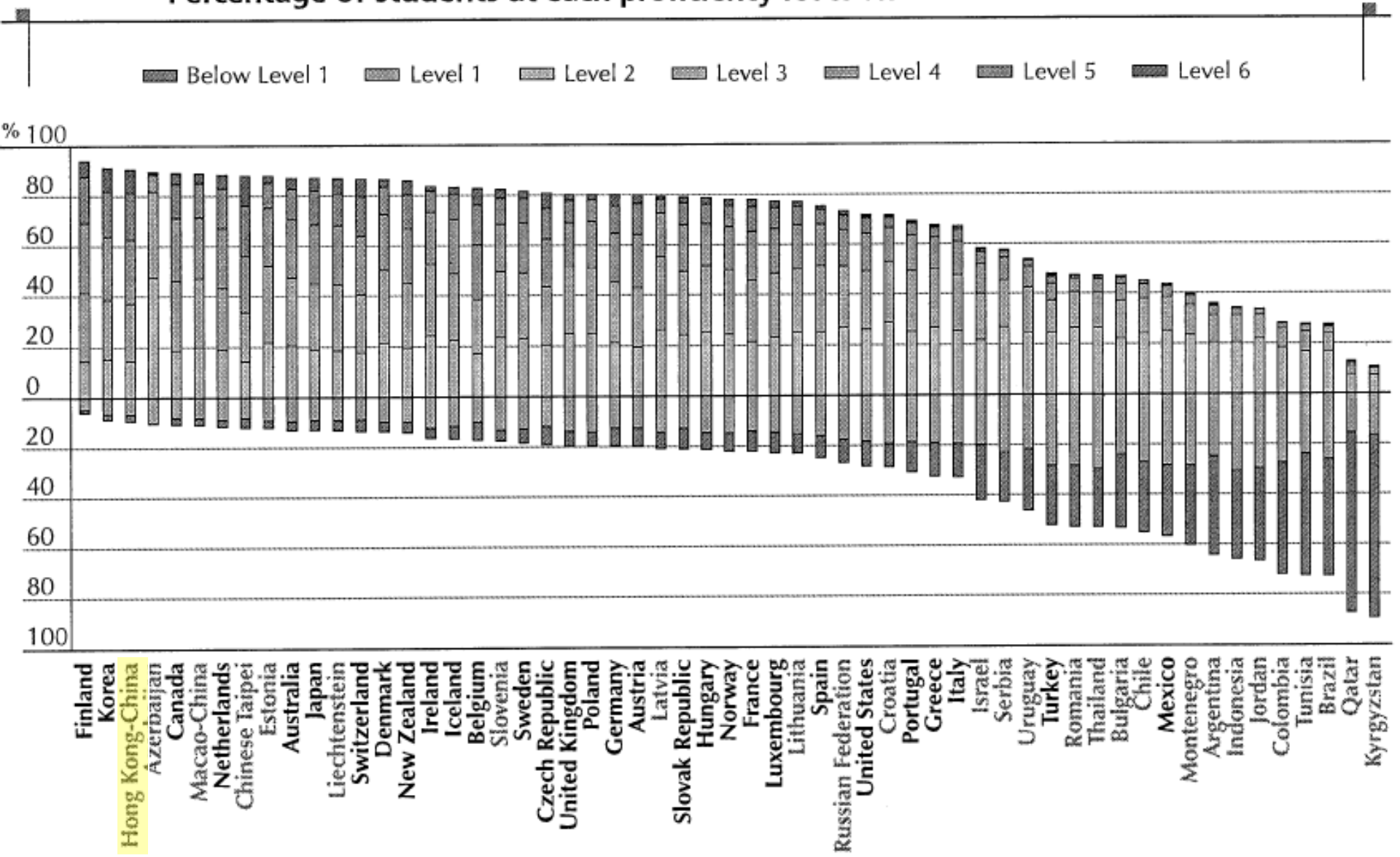


Countries are ranked in descending order of percentage of 15-year-olds in Levels 2, 3, 4, 5 and 6.

Source: OECD PISA 2003 database, Table 2.5a.

Figure 6.19

Percentage of students at each proficiency level on the mathematics scale

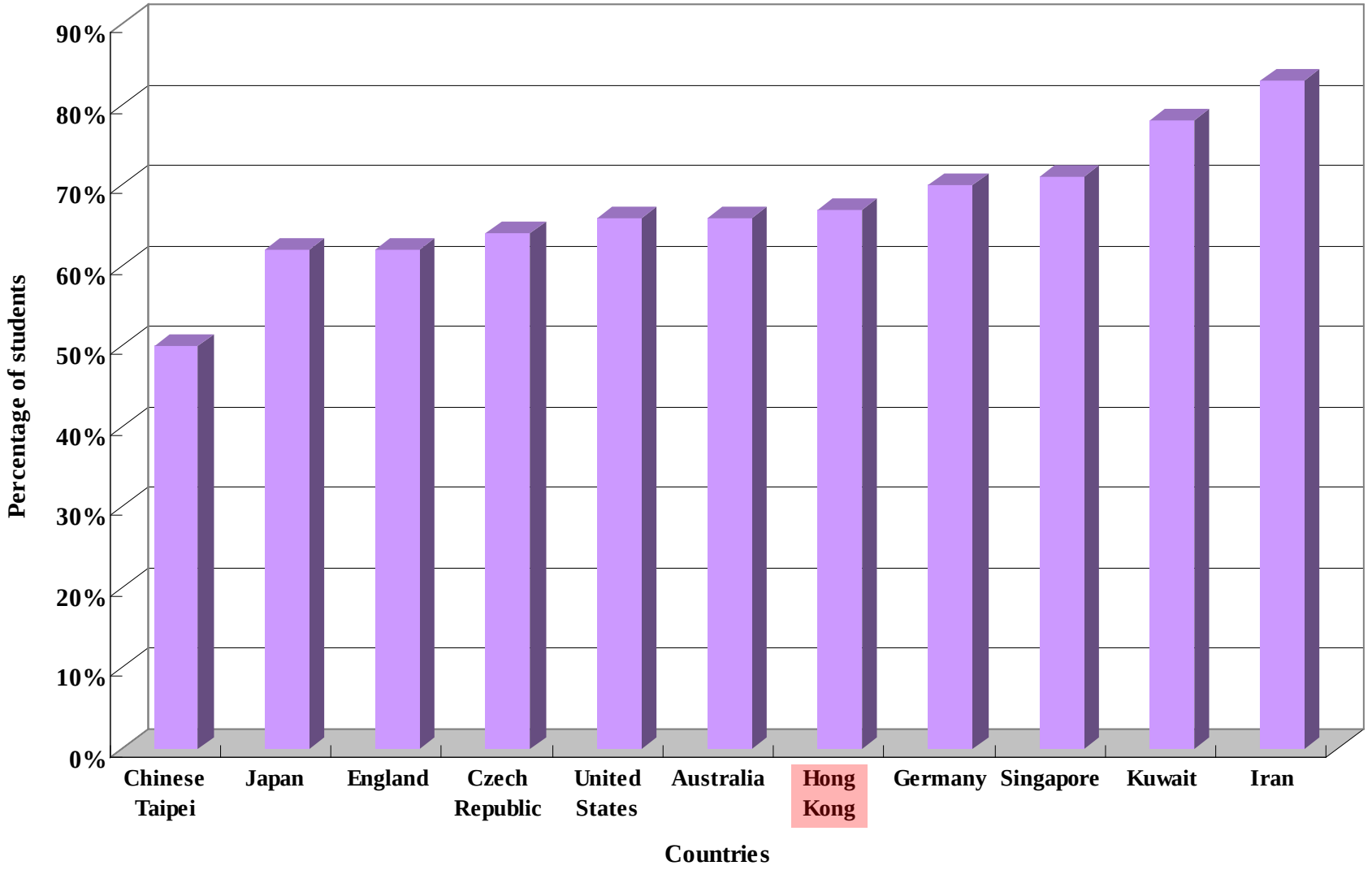


Countries are ranked in descending order of percentage of 15-year-olds at Levels 2, 3, 4, 5 and 6.

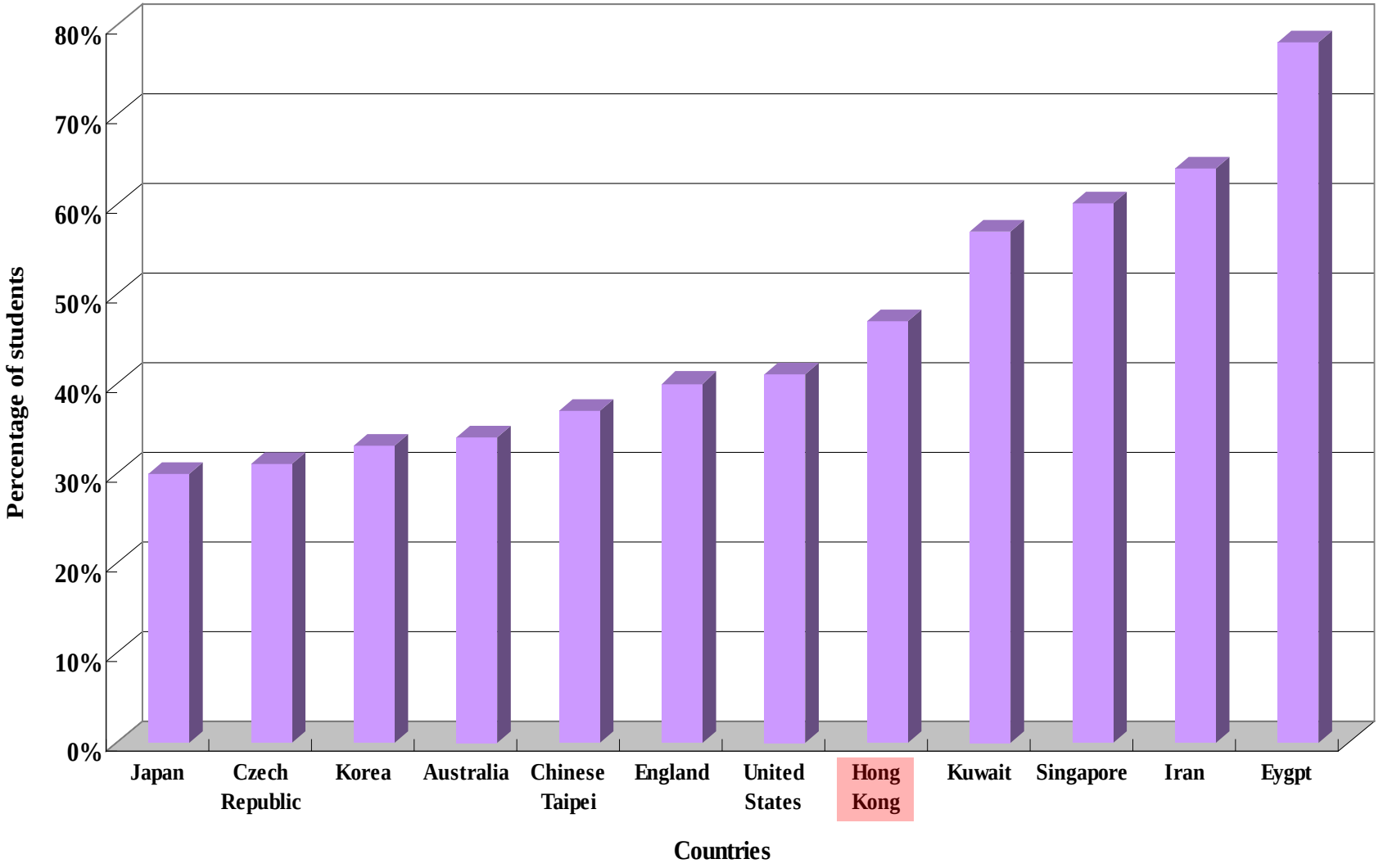
Source: OECD PISA 2006 database, Table 6.2a.

StatLink http://dx.doi.org/10.1787/142046885031

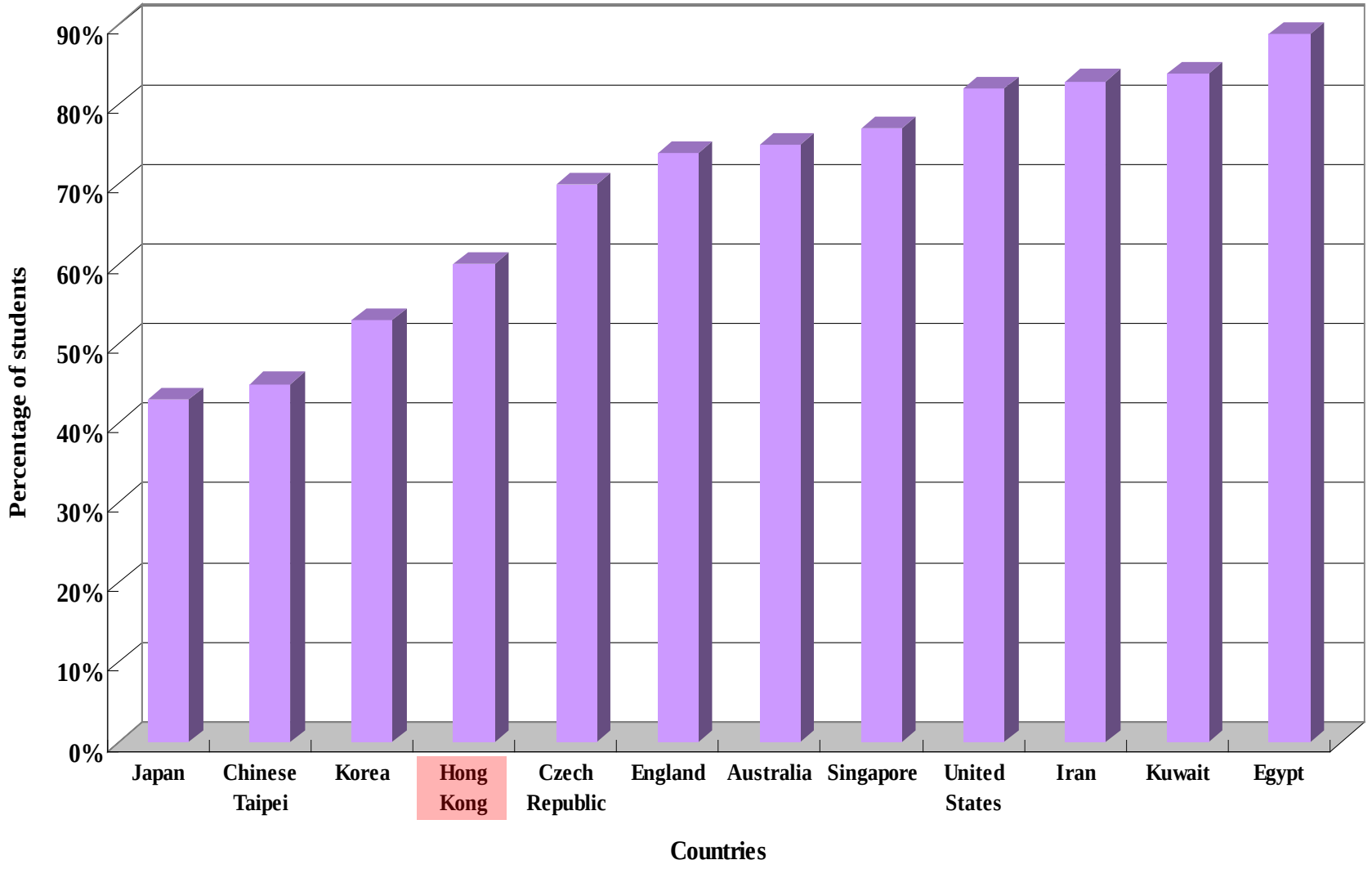
High Positive Affect Toward Mathematics (G.4 Int'l Avg. = 72%)



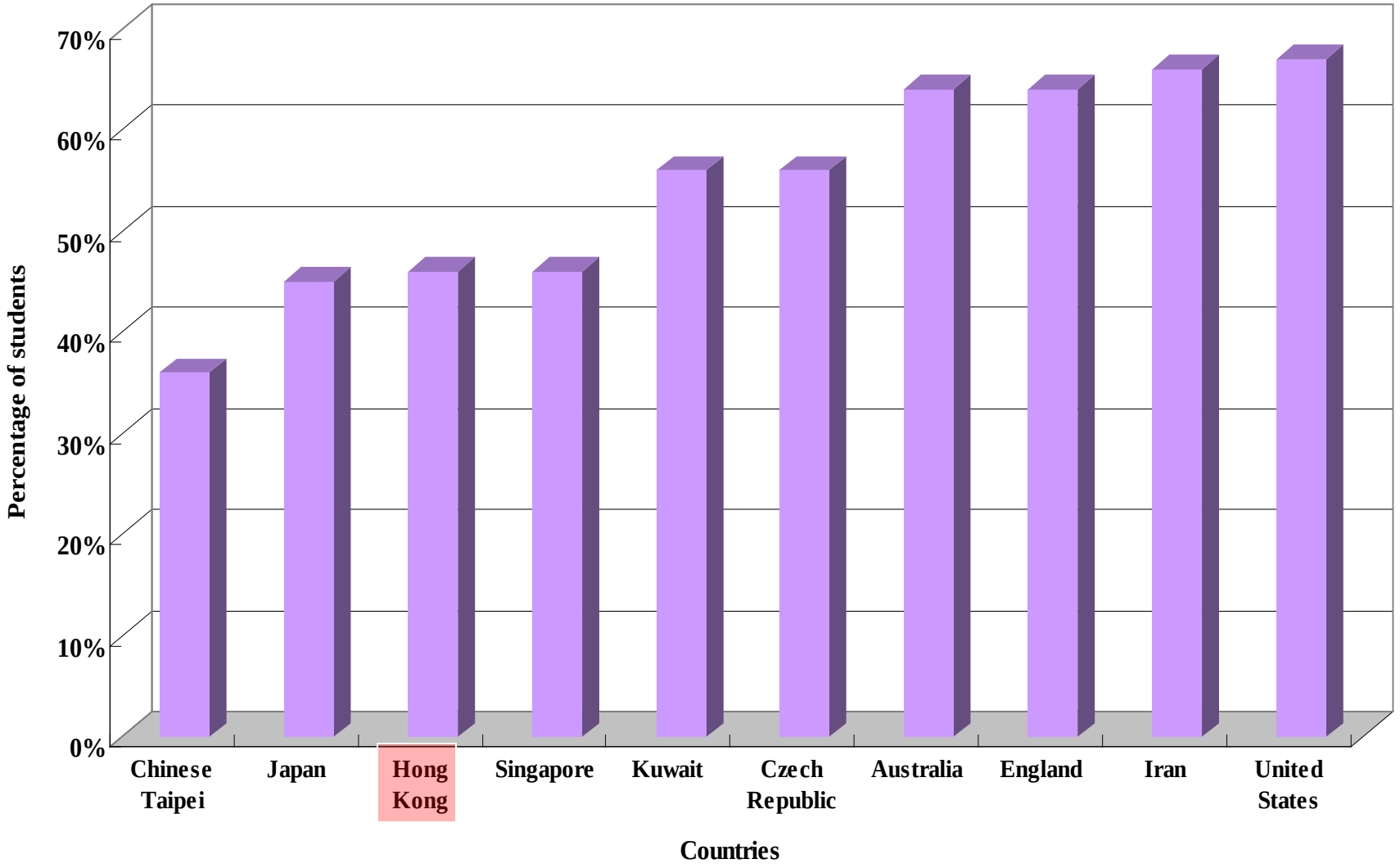
High Positive Affect Toward Mathematics (G.8 Int'l Avg. = 54%)



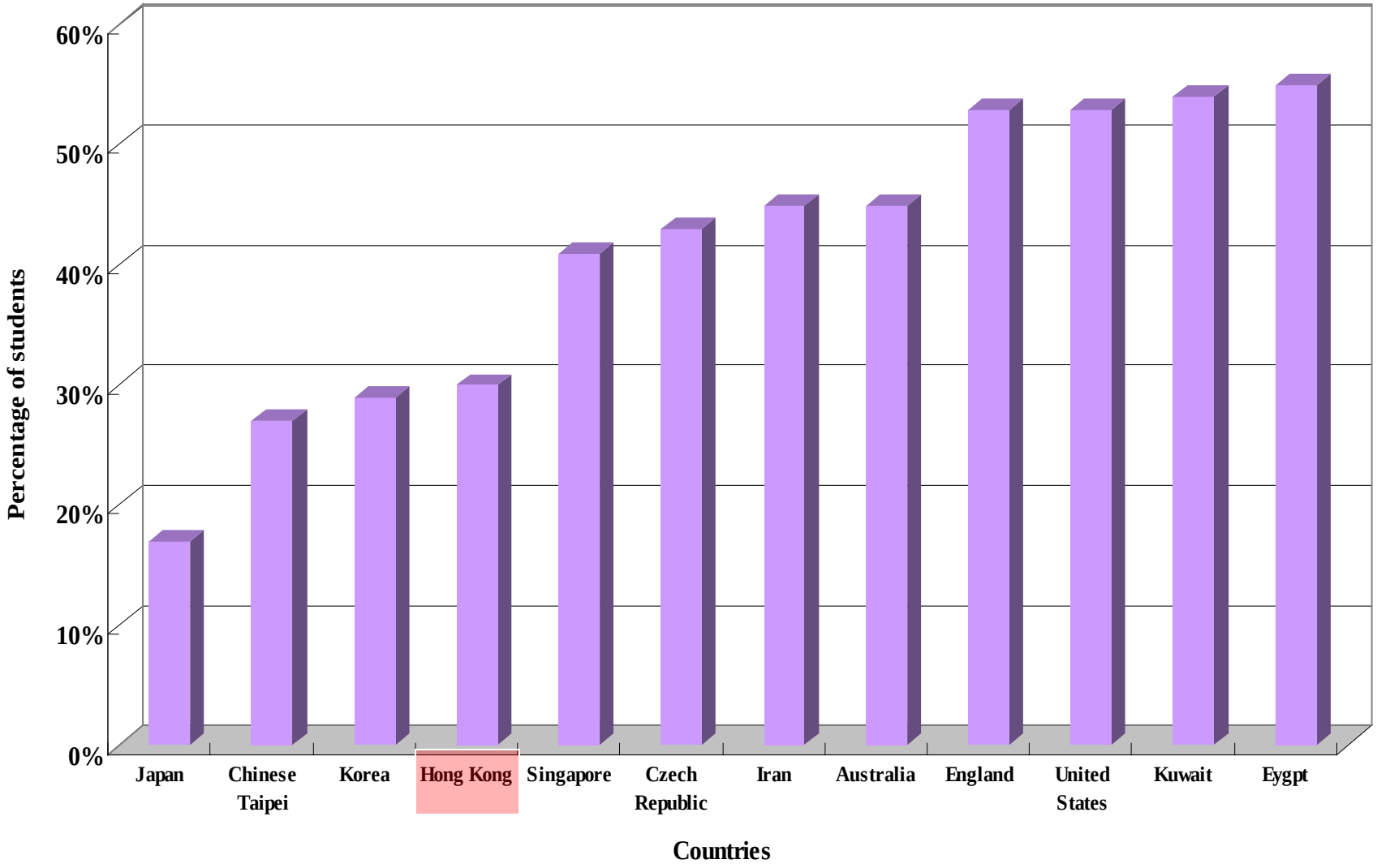
Students Valuing Mathematics Highly (G.8 Int'l Avg. = 78%)

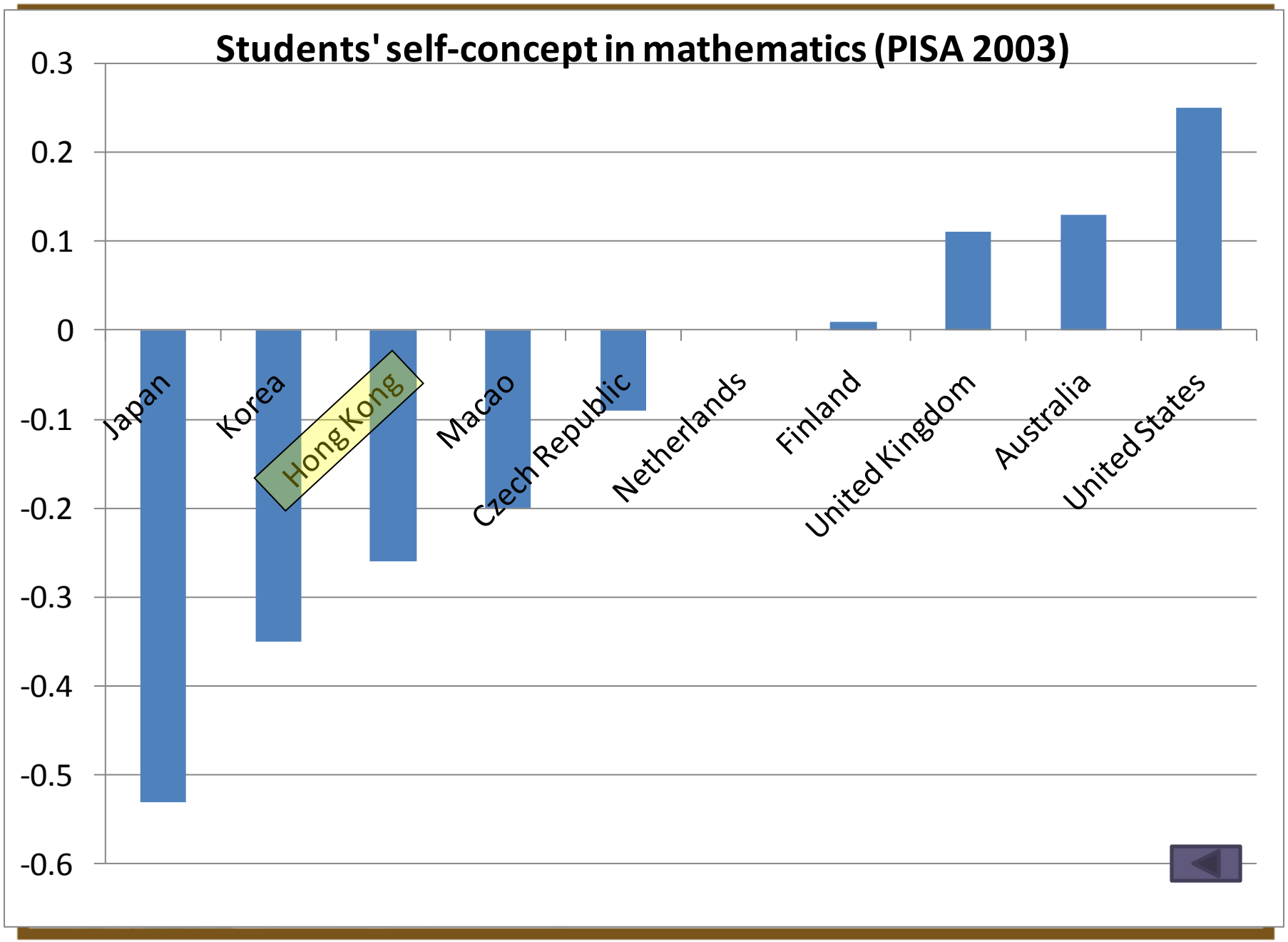


High Self-Confidence in Learning Math. (G.4 Int'l Avg. = 57%)



High Self-Confidence in Learning Math. (G.8 Int'l Avg. = 43%)




Students' self-concept in mathematics (PISA 2003)
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
Japan
Korea
Hong Kong
Macao
Czech Republic
Netherlands
Finland
United Kingdom
Australia
United States

Nunes 的「*街頭數學*」研究

- 時間: 1982年
- 地點: 巴西城市Recife (人口150萬); 有大量附近農村來的民工
- 小孩(綴學或放學後)幫助父母在街上做買賣非常普遍
- 訪談了5個九到十五歲的小孩(4男1女; 平均年齡11.2歲; 學歷小一到初中二)
- 方法: 街頭訪談;數天后, 跟據訪談的內容進行筆紙的測試

訪談內容舉例

(I = 研究員; M是一個12歲,三年級學歷的男孩)

I: 一個椰子多少錢？

M:三十五元

I: 我要買十個;多少錢？

M: (稍停頓)三個是一百零五;再三個是二百一十(稍停頓)我再需要四個,這是… (稍停頓)三百零十五…這是三百零五十

分析:

M懂得算: (1)35x10; (2)35x3(一早已知的結果); (3)105+105; (4)210+105; (5)315+35; (6)3+3+3+1

訪談與筆紙測試

I:　現在我要買四個;多少錢？

M:那是一百零五,加三十,那是一百三十五…一個椰子是三十五…那是…一百四十

測驗:35x4

M:四乘五是二十,進位二,二加三是五,乘四是二十

M寫下答案: 200

5個小孩的測試結果:

街頭題98.2%; 文字題73.7%; 計算題36.8%

華爾街日報(2009年一月六日)

The Best and Worst Jobs

1. Mathematician
2. Actuary
3. Statistician
4. Biologist
5. Software Engineer
6. Computer Systems Analyst
7. Historian
8. Sociologist
9. Industrial Designer
10. Accountant

200. Lumberjack
199. Dairy Farmer
198. Taxi Driver
197. Seaman
196. EMT
195. Roofer
194. Garbage Collector
193. Welder
192. Roustabout
191. Ironworker

The Best Jobs

Wall Street Journal, January 6, 2009

"The study, released Tuesday from CareerCast.com, a new job site, evaluates 200 professions to determine the best and worst according to five criteria inherent to every job: environment, income, employment outlook, physical demands and stress. The findings are based on data from the U.S. Bureau of Labor Statistics and the Census Bureau, as well as studies from trade associations."

"According to the study, mathematicians fared best in part because they typically work in favorable conditions - indoors and in places free of toxic fumes or noise - unlike those toward the bottom of the list like sewage-plant operator, painter and bricklayer. They also aren't expected to do any heavy lifting, crawling or crouching - attributes associated with occupations such as firefighter, auto mechanic and plumber. The study also considers pay, which was determined by measuring each job's median income and growth potential. Mathematicians' annual income was pegged at $94,160."

丘成桐 訪問

Discover (Magazine), June 2010
(published online Tuesday, April 27, 2010)

Interviewer: When you looked at the world through the lens of geometry and topology, what did you learn?

Yau: That nonlinear equations were fundamental because in nature, curves abound. Climate isn't linear. If the wind blows stronger that way, it may cause more trouble over there; it may even depend on the geometry of the earth. Usually you see the stock market described by linear equations and straight lines, but that is not really correct. The stock market fluctuates up and down in a nonlinear way. The Einstein equation described the curvature of the universe, and it was nonlinear.

I: Why should anyone other than a mathematician care about a torus or a string hidden within higher dimensions?

Yau: Because topology can affect and constrain geometry in the physical world. If water flows around a sphere, for example, there must be two points where the water is totally still. On a planet covered with an ocean, the water can't all flow in the same direction, say east to west, everywhere, without hitting a snag. In the case of another topology, the torus, water can flow around and around and there's no point at which the flow stops because the hole eliminates the impasse. For each fixed topology, the geometry follows different laws.

Interviewer: In other words, you realized that topology sets the basic rules for geometry, which in turn affects the world around us. But then you went further, asking whether the underlying structure of space might explain the laws of physics. How so?

Yau: I started to look into complex manifolds. A manifold is just a space, with each point immediately around you looking like Euclidean space-the familiar kind of space that we see around us. Imagine the earth is covered with a checkerboard or a grid, like latitude and longitude. This is the kind of coordinate system that Descartes introduced to geometry in the 17th century. At each point on the grid the space appears flat and finite, but it's actually curved, a sphere. Instead of being measured with real numbers, though, we measure complex manifolds with complex numbers, in which one of the coordinates includes a real number multiplied by the square root of negative 1-an imaginary number that we call i.

Extracts from Stobaeus

A youth who had begun to read geometry with Euclid, when he had learnt the first proposition, inquired, "What do I get by learning these things?" So Euclid called a slave and said "Give him three pence, since he must make a gain out of what he learns."

例 2

因式分解下列各式：

(a) $x^2 + 3x - 18$；

(b) $x^2 + 7x + 12$。

可改寫爲：

爲使下式能以因式分解，a 和 b 分別可取哪些整數值?

(a) $x^2 + ax - 18$；

(b) $x^2 + 7x + b$。

例 3

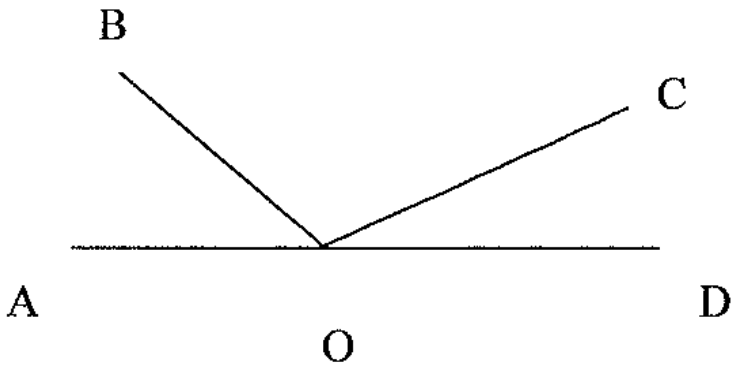
求圖中鈍角的數目。

(a) 1

(b) 2

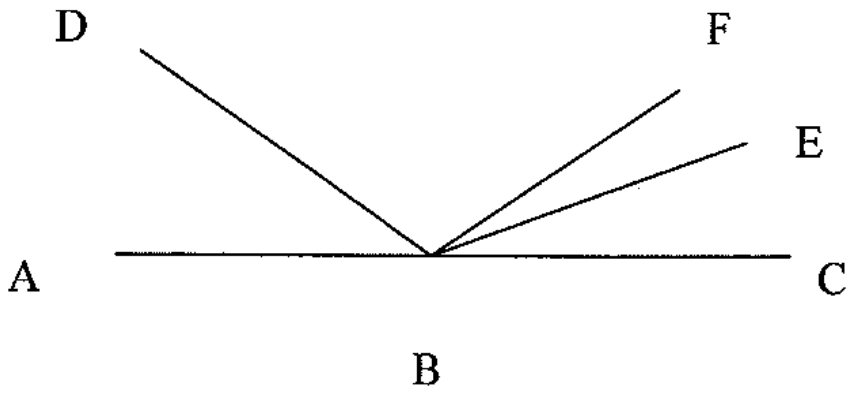
(c) 3

(d) 4



可改寫爲：

任意寫出下圖兩個鈍角的名稱。



例 4

五個女孩和三個男孩分別收到 18、20、19、21、22、23、24 和 29 張聖誕咭。求八人平均收到咭的數目。

可改寫爲：

五個女孩和三個男孩收到友人的聖誕咭。五個女孩收到咭的平均數目爲 20 張。三個男孩收到咭的平均數目爲 28 張。句子『收到最多咭的一定是一個男孩。』是否真實？試解釋。

例 9（統計與真相）

在 2000 年，香港有 100 宗消費者投訴物業代理佣金過高的個案。消費者委員會在這一年起訴了其中 2 宗。在 2001 年有 300 個這樣的個案，結果有 3 宗被起訴。而消費者委員會主席不打算起訴其他投訴的個案。

陳大文先生是 2000 年和 2001 年的主席。他正參加改選。競爭對手是潘嘉莉女士。

潘女士在最近一次的競選活動指出：「現任主席在打擊物業代理對顧客的不公允一事上紀錄欠佳。他在 2000 年只起訴了百分之二被舉報的個案，在 2001 年只起訴了百分之一。請投我的票，我可以使情況得到改善，並將會嚴厲執行法例。」

這是陳先生的回應：「我已努力在物業代理佣金過高一事上維護顧客本身的權益。雖然財政緊拙，我們已改善了起訴的紀錄。在 2001 年，我們就佣金過高的起訴較對上一年增加 50%。請投我一票，我將會在香港繼續改進法例的執行。」

不用理會個人的主張，單就數學的概念而言，那一位侯選人的言論正確？試加解釋。

例 10

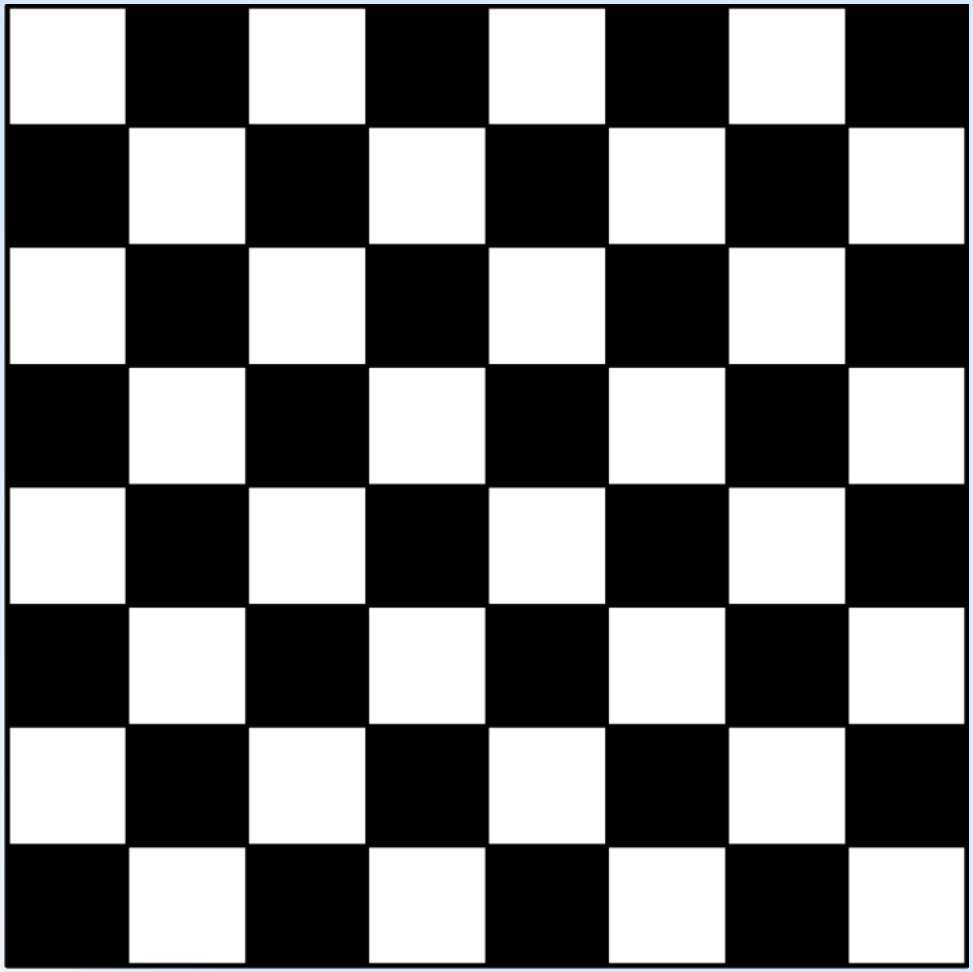
香港青年會的委員每星期均舉行會議。在三月，會議最多能有多少次？最少能有多少次？

扼殺創新力十著

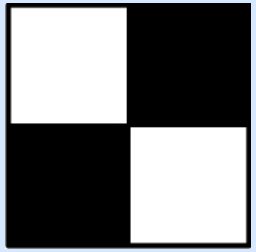
1. *必須用得著的才做*
2. *必須成功的才做*
3. *要十全十美才做*
4. *要受人喜歡*
5. *不可離群獨處*
6. *切記集中注意力並視之為神聖*
7. *切勿脫離既定的性別規範*
8. *不可表露過多情感*
9. *不能含糊不清楚*
10. *不能搞亂已有規律*

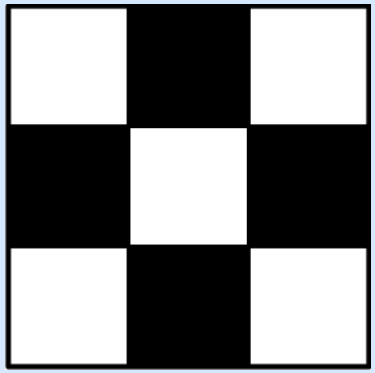
Krippner, S. (1967). The ten commandments that block creativity. *The Gifted Child Quarterly, 11*, 144-151

下圖共有多少個正方形?

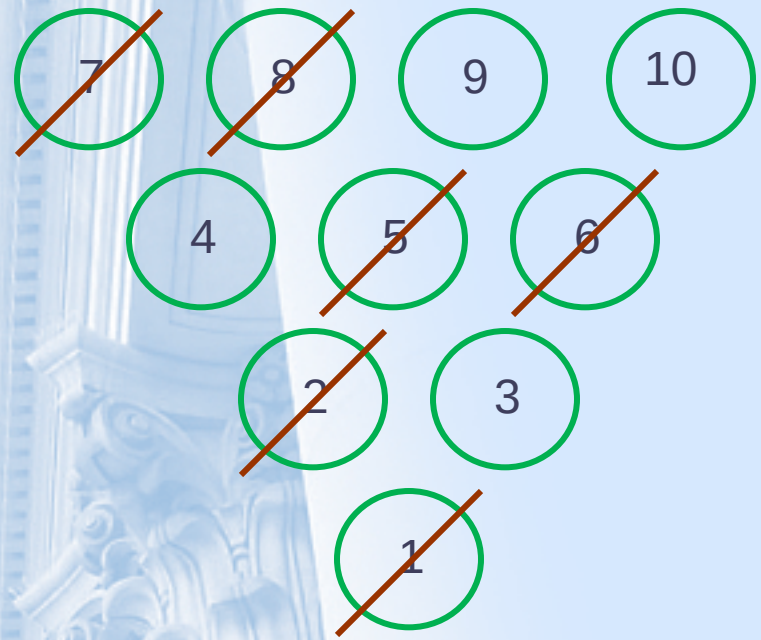


下圖共有多少個正方形?





數字保齡球



1 = 3+4-6

2 = (3×4)÷6

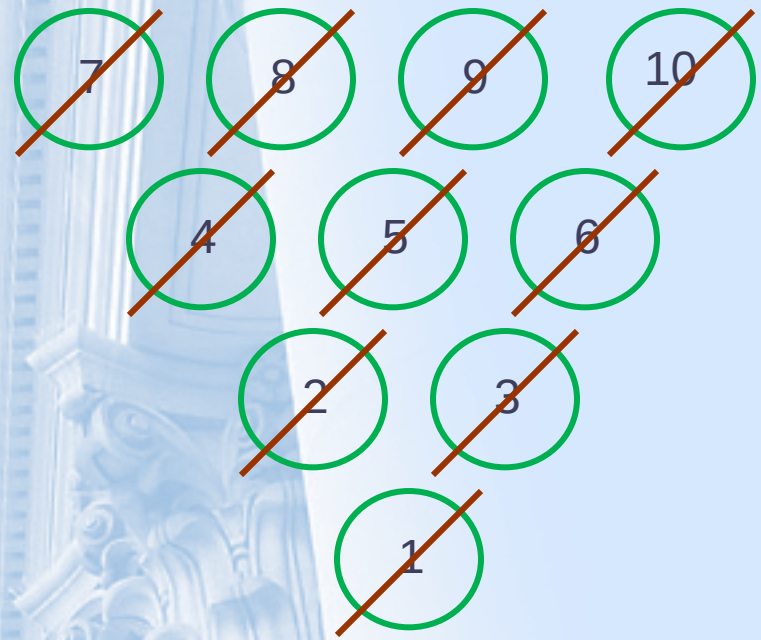
5 = 6-(4-3)

6 = 6÷(4-3)

7 = 4+6-3

8 = 4×(6÷3)

例如: 擲三粒骰子結果是 3, 4, 6



3 = 4-(3÷3)

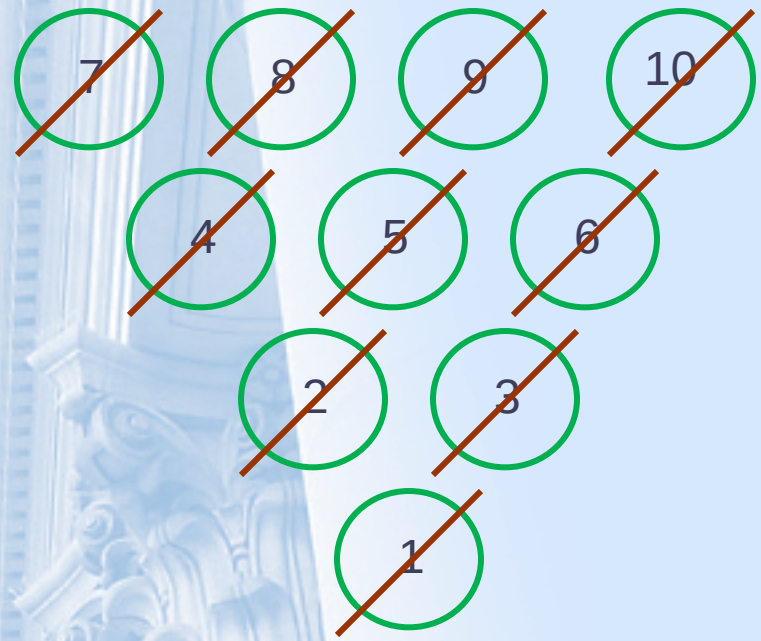
4 = 4×(3÷3)

9 = 3×4-3

10 = 3+3+4

頂呱呱!!

第二次擲骰子結果是 3, 3, 4



$4 = (6/3) + \sqrt{4}$

$3 = 6 - [\sqrt{3} + \sqrt{2}]$

$9 = 3 + 4 + [\sqrt{6}]$

$10 = 3 \times 4 - [\sqrt{6}]$

頂呱呱!!

例如: 擲骰子結果是 : 3, 4, 6